“Success is a **journey** not a destination” – Arthur Ashe, Tennis player (seen in Valedictorian speech on TV)

"It is not knowledge, but the act of learning, not possession but the act of getting there, which **grants the greatest enjoyment**" – Carl Friedrich Gauss, Mathematician

“It is a **collective journey**, one of the **most ancient and powerful** ever. Begun at the dawn of time by a handful of human beings, it continues to this very day to **transform** our **civilization**, **language**, and **thought**” – Mathematica by David Bessis, Mathematician [8]

"The ideals which have always shone before me and filled me with joy are **goodness**, **beauty**, and **truth**." – Albert Einstein

“Even though we all had sort of different ideals as to what good mathematics should look like, they all kind of tend to **converge** to the same thing” – Terence Tao [2]

"Truth is good and beautiful; goodness is true and beautiful; beauty is true and good." – C.S Lewis

“We shall not cease from exploration, and the end of all our exploring will be to arrive where we started and know the place for the first time.” - T. S. Eliot

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Why Mathematics

Beauty [2]

<u>Concise</u>

- Unification
  - Definition: Ability to connect and consolidate otherwise separate concepts
  - Motivation: Understanding
    - Analogy: **Blind men initially describing an elephant differently** (one touches the trunk, the other touches the tusks, etc) only to **later find out that they were describing the same elephant**
  - Example: Different possible nets all come from the same 3D shape
  - Example: Different possible curves come from the same ice cream cone conic shape
  - Example: Euclid was able to explain the properties of circles and quadrilaterals in terms of the properties of triangles, which in turn were explained using the properties of lines and angles
  - Example in Physics: Maxwell unified electricity and magnetism in 4 equations
- Simplicity
  - Definition: Elegance in the sense of requiring the fewest assumptions/axioms
  - Example in Physics: Different models of what matter is made of (atoms) required fewer assumptions over history (for example

the shape of orbitals and number of electrons in each shell were explained by later equations)
        - Caveat: Even though they require learning some new words and ideas to express oneself (for example 2D complex numbers, differential equations)
            - This may require building new intuition but it is worthwhile because of its elegance
    - Example in Physics: Maxwell unified electricity and magnetism in 4 equations
- Epistemic Tractability
    - Definition: Quality of being cognitively accessible. Allowing the concept to be clearly surveyed, grasped and understood by the human mind.
    - Example: Kac-Erdos overview of proof of normal distribution of prime number involves 4 step outline [14]
    - Example: Proof for the existence of infinite prime numbers, or the irrationality of root 2 is short enough to be grasped.

<u>Consistent</u>

- Explanatory Depth
    - Definition: Ability to explain all theorems axiomatically via proof
    - Example: (Also relevant here) Euclid was able to explain the properties of circles and quadrilaterals in terms of the properties of triangles, which in turn were explained using the properties of lines and angles
    - Example in Physics of Explanatory Depth: Description of atomic orbital shapes using Schrodinger's equation
    - Example in Physics of Precision Prediction: Comparisons of hyperfine transition frequencies with prediction from Quantum Electrodynamics
- Generality
    - Definition: The power of theory that comes from abstracting away irrelevant details to retain just enough information to solve a problem (show it is a special case of a more general problem that has already been solved or shown to be unsolvable) or determine its solvability/truth.
    - Example: Galois theory proved the insolvable nature of roots of 5th degree polynomials by reframing polynomials in the more abstract context of group theory, the *"natural setting"* for this problem where solvability becomes clear.

        - [15] This is the essence of “clever theoretical work”.

<u>Complete</u>

- Discoverable by Theory Alone
    - Definition: The quality of a result being discoverable only through clever theoretical work [15] (abstraction/generalization), not by experiment or induction.
    - Example: Proof that there are an infinite number of prime numbers (was not discovered by counting them manually).
    - Example: Kac-Erdos theorem proof for distribution of prime factors of all numbers following a bell curve (beyond current computational power).
- Surprisingness
    - Definition: Ability to explain established ideas in new, non-obvious and counter-intuitive ways that overturn prior expectations.
    - Example: Discovery of irrational numbers like root 2 showed that not all numbers could be written as simple fractions.
    - Example in Science: Discovery that most of the atom is empty space was counter-intuitive compared to the “solid ball” model.
- Fruitfulness
    - Definition: Ability to predict new discoveries, insights, research questions
    - Example in Physics: The Dirac equation predicted the existence of antimatter [9]
    - Example in Chemistry: Mendeleev predicted the existence of missing elements in the periodic table
        - 2019 was dubbed by UNESCO as the Year of the Periodic Table [10]
    - Example in Biology: The Germ Theory of Disease led to sterilization, antibiotics and vaccines.

Problem Solving [2]

<u>Complete</u>

- Difficulty
    - Motivation: It is done for the same reason that some people climb mountains (for immortalizing achievement) and other people climb stairs (for mental fitness)

Consistent

- Rigor
    - Motivation: Certainty: Certainty in logical rigor

Concise

- Elegance
    - Motivation: Understanding: The joy of finding out how something initially complicated is simple

Applications [2]

- Reveals **Truth**
    - Motivation from completeness: Truth: The discovery of truths that follow from assumptions chosen so that conclusions match reality
    - Motivation from consistency: Full certainty follows from proofs and proofs are possible only in mathematics.
    - Motivation from human interest: Permanence: A proof, once verified, is eternal. What was proved by the Ancient Greeks thousands of years ago remains valid today.
    - Example: Proof of existence of infinite number of prime numbers
    - Example: Proof or irrationality of square root 2

Consistent

- Finds interconnections
    - Example: The Langlands program links different branches of mathematics
        - Focus: On understanding fundamental “mother-structures” like number, functions of complex variables and abstract algebra through algebraic number theory, the theory of automorphic forms and the representation theory of algebraic groups.
        - Example: Proof of Fermat’s last theorem by Andrew Wiles relies on one part of the Langlands program
    - It is difficult to tell what is useless [1] in the long run [7]
        - Science: Esoteric mathematics often turned out to be useful in science [3], a popular view is that the universe is mathematical (eg: June Huh’s paraphrased “We are discovering interconnected mathematical structures that

feel like discovering external reality" and Tegmark's Mathematical Universe Hypothesis)
    - Quote: "Beauty is truth, truth beauty,—that is all. Ye know on earth, and all ye need to know." - Keats
  - Economics: What was initially invented for pure science later became useful in economics
    - Example: Calculus and the Black Scholes equation

Human Interest

- Motivation: Allows one to develop **good character virtues** [12] **transferrable** to other areas of life
- Motivation: Is intrinsically **fun and surprising** [5][11] to do, individually and as a team [13]
  - Motivation: In the sense that **finding out what is true and why it is true** satisfies curiosity, need for seeing the overall beautiful picture and need for certainty.
  - Analogy: It is like building a jigsaw puzzle and being satisfied in finding that the pieces snap together to reveal the bigger, more beautiful picture
  - Analogy: It is like appreciating the forest after observing trees.
- Motivation: It allows one to extend one's time in school **meaningful**ly [5][6]
- Motivation: Is a common / universal topic that **bridges cultural differences**
  - Definition: It is universal in that it is the same throughout the universe [4] and agreed across cultures.

Accessibility

- Motivation: Is cheaper to fund than the experimental sciences!
  - Does not require any special lab equipment
    - Pen, paper, blackboard
    - Personal computer, printer
    - Cloud server compute time

Background: Brief Biographies

Bourbaki

- Pen pseudonym for a group of mathematicians that publishes in attempt to make math more beautiful.

- Interested in studying “mother structures” that originally came from desire to describe some feature of the natural world in a mathematical, abstract way: “natural numbers, the real line, the Euclidean plane, topological structures, order structures and algebraic structures”.

Terence Tao

- Fields Medal winner, 2006
- Breakthrough Prize winner, 2014

Timothy Gowers

- Fields Medal winner, 1998
- Author of Mathematics, A Very Short Introduction and The Princeton Companion to Mathematics (Won the MAA’s Euler prize)

June Huh

- Failed math in primary school, dropped out of secondary school at age 16, took 6 years to complete 4 year undergraduate physics degree with intention to become science journalist due to maths difficulties
- Fields Medal winner 2022

David Hilbert

- Attempted to axiomatize mathematics.
- Announced 23 problems at the frontier of mathematics in 1900 that became the basis of the Millennium prize.

Gregori Perelman

- Declined the Millennium prize because it was not offered to the person whose work he built upon.

Daniel Waxman

- Harvard double first in Mathematics and Philosophy
- Current NUS professor (wrote Is Mathematics Unreasonably Effective before ChatGPT was released, while at Lingnan university before moving to NUS)

Francis Su

- Past president of the Mathematical Association of America
- Author of Mathematics for Human Flourishing

Yuri Milner

- Author of the Eureka manifesto.
    - #Todo: Summarize significance of Eureka manifesto, the manifesto that aims to contact intelligent life in space in order to unify and inspire intelligent life on Earth.

Landon T. Clay

Martin Bridson

Andrew Wiles

Background: Prizes and Organizations

Fields Medal

- Harder to win than the Nobel prize because it is only awarded to those below 40.
- Worth $15,000 CAD, second to the Abel prize (closer to $1 million dollars)
- Started in 1936, only 64 people have won it (around 3 people every 4 years).
- Medal states “To surpass one's understanding and master the world”.
- Is famous because of the film Good Will Hunting, where professor Lambert won the medal before the events of the film.

Breakthrough Prize

- $3 million dollar annual prize given in mathematics, fundamental physics and life sciences. Funded by Jack Ma, Sergey Brin, Mark Zuckerberg, Yuri Milner.

Millennium Prize

- $1 million dollar prize for frontier problems
- None have been solved except the Poincare conjecture. Gregory Perelman proved the conjecture but turned down the prize in 2010.

Mathematical Association of America

- Runs the Meet a Mathematician Youtube channel, with speakers like past president Jenny Quinn who wrote an article What is Mathematics for MAA Focus (the association’s newsletter).

Clay Mathematical Institute

- Started in 1998 by Landon T. Clay to “further the beauty, power and universality of mathematical thought”.
- Funds the Millenium prizes prize money.

International Mathematical Olympiad

- Started in 1959. The brainchild of Romanian mathematician Grigore Moisil, who proposed the competition.
- Initially organized by the Romanian Mathematical Society and involved seven countries from the Eastern Bloc: Bulgaria, Czechoslovakia, East Germany, Hungary, Poland, Romania, and the Soviet Union.
- 7 problems (max score 42) from algebra, geometry, combinatorics and number theory. Solutions are often elementary.
- Contestants must be under the age of 20 and not registered at any tertiary institution.
- Awards are given to half the participant. Gold:Silver:Bronze awards are given out in the ratio 1:2:3.
- Google Gemini 2.5 Deep Think won IMO Gold in July 2025. [16]

Simons Foundation

- Funds Quanta Magazine, a popular math and science magazine that maintains editorial independence. Quanta Magazine also hosts the Joy of Why podcast.

References (Read)

Terence Tao – What is Good Mathematics – Quanta The Joy of Why (Article & Podcast)

https://www.quantamagazine.org/what-makes-for-good-mathematics-20240201/ https://archive.is/RDg5Y

[2] Beauty, Problem Solving and Applications are 3 paths that converge to the same “Good mathematics” mountaintop view

“Like, some would pursue **applications**, some by sort of aesthetic **beauty**, some by just **problem solving**. They wanted to solve a problem and they would focus on sort of the most *difficult*, the most challenging tasks. Some would focus on technique; some would try to make things as *elegant* as possible.

> But what struck me when sort of listening to so many of these different mathematicians talk about what they find valuable in mathematics is that, even though we all had sort of different ideals as to what good mathematics should look like, they all kind of tend to **converge** to the same thing."

[13] A greater percentage of math papers are nowadays being written by more than 1 author due to the length of the proof involved, and need for cross-topic expertise.

Daniel Waxman – Is Mathematics Unreasonably Effective? (Published Paper)

https://danielwaxman.com/files/applicability.pdf

https://archive.is/Q55jj

[3] Definition of beauty in science shares many things in common with definition of beauty in mathematics, hence what is pursued in math later turns out to be useful in science

[9] Example of Dirac equation as something derived for theoretical beauty (unification of special relativity and quantum mechanics, symmetry) that turned out to be useful in science (in the prediction of antimatter before it was discovered)

[14] Per page 14's restatement of Gower's outline #Todo: Copy excerpt and link to Timothy Gowers' Importance of Mathematics

Timothy Gowers – Importance of Mathematics (Written Speech)

https://www.dpmms.cam.ac.uk/~wtg10/importance.pdf

https://archive.is/3rMaU

Available on YouTube https://www.youtube.com/watch?v=BQHbdi8A0p8 (linked via the Riemann Hypothesis' related videos on the Clay Mathematical Institute page)

[1] It is hard to determine what is useless in mathematics because initially no-applications purely-theoretical explorations later turn out to be useful in science

[7] Example of mathematics that turned out to be useful in the longrun: Non-Euclidean geometry turned out to be useful for general relativity

Outliner 1.6.8: Section 3 The Economic Argument: An Interconnected Web of Knowledge, sub-section 3.1 The Unpredictability of Useful Mathematics

#Todo: Attach Outliner 1.6.8 link

Francis Su – Mathematics for Human Flourishing (Book)

[12] https://nlb.overdrive.com/media/5180742

James Robert Brown – Philosophy of Mathematics 2E (Book)

#Todo: Attach outliner 1.6.8 summary and NotebookLM audio overview

Discovered from: PhilosophyNow's Massimo Pigliucci https://philosophynow.org/issues/84/Mathematical_Platonism

Chp 1

Examples of the mathematical image (commonly held beliefs about maths)

- Existence of infinite number of Primes
- Irrationality of square root 2

References (Reading)

[8] David Bessis – Mathematica – Quora Magazine – "Mathematical Thinking Isn't What You Think It Is"

https://www.quantamagazine.org/mathematical-thinking-isnt-what-you-think-it-is-20241118/

https://archive.is/hboYz

Simon Singh - Fermat's Enigma

Epilogue

References (Not used)

David W. Hansen – The Dependence of Mathematics on Reality

Seen in Knowledge and Inquiry file handout

Published as part of an anthology in “Five Fingers to Infinity” book

https://www.ebay.com/itm/363023752493

https://archive.is/18AOO

Originally published in The Mathematics Teacher, a magazine for math teachers

References: YouTube Videos

June Huh – What motivates us is the pursuit of beauty [3 mins 42 seconds]

https://www.youtube.com/watch?v=A4SJChFuibY

[5] Surprise and beauty are sources of enjoyment

[6] Doing mathematics feels like discovering an external reality

Steven Strogatz – Why Learn Math [2 minutes 40 seconds]

https://www.youtube.com/watch?v=d2hpkI9kSlI

[11] It is like the joy of solving puzzles as a little kid, such as solving jigsaw puzzles

Edward Witten – How is Mathematics Truth and Beauty [6 mins 36 seconds]

https://www.youtube.com/watch?v=O3isFuQ2q2A

[4] Aliens would have discovered calculus if they were curious about the relationship of volume to surface area of a sphere

YouTube search leads:

- Why is math truth and beauty
    - PBS Closer to Truth episodes and interviews
- MAA Meet a Mathematician

References: Excerpts

[10] https://www.aps.org/archives/publications/apsnews/updates/iypt.cfm

> "All but two trans-uranium elements, from Neptunium (93) to Oganesson (118), have been discovered at cyclotron accelerator facilities in the US, USSR/Russia, Germany, and Japan."

[16] https://deepmind.google/discover/blog/advanced-version-of-gemini-with-deep-think-officially-achieves-gold-medal-standard-at-the-international-mathematical-olympiad/

> "We can confirm that Google DeepMind has reached the much-desired milestone, earning 35 out of a possible 42 points — a gold medal score. Their solutions were astonishing in many respects. IMO graders found them to be clear, precise and most of them easy to follow." - IMO President Prof. Dr. Gregor Dolinar

References: Want to Read/Reread

Mathematics A Very Short Introduction – Timothy Gowers

Princeton Companion to Mathematics

Fermat's Enigma – Simon Singh

Journey Through Genius – William Dunham

History of Mathematics – Carl Boyer

Mathematica – David Bessis

Philosophy of Mathematics 2E - James Robert Brown

Mathematics Rebooted – Lara Alcock

How to Study for a Mathematics Degree – Lara Alcock

Mathematics for Human Flourishing – Francis Su

Changelog

2 Aug 2025

- Version 0.33 9:33-10:34pm
    - Removed chemistry full valence shell rule as example of explanatory depth.
        - Because it is alone insufficient to explain exactly which of various possible products will form (there are rules from organic chemistry that are also required for explanatory depth, for example Markovnikov's rule for alkene substitution)
    - Added International Mathematical Olympiad to Background: Prizes and Organizations.
        - Discovered Kiddle.co (Wikipedia for Kids)
        - Discovered Timothy Gower's Importance of Mathematics speech was uploaded to YouTube by the Clay Mathematical Institute in 2022 (presently has 7k views).
        - History of IMO is not yet on Wikipedia (its founder and Google's Gemini 2.5 Deep Think's Gold Medal is not mentioned yet, found with AI Studio requires some fact checking).
    - Added History of Mathematics by Carl Boyer to References: Want to Read/Reread

18 July 2025

- Version 0.31 10:06pm Added beauty, goodness, truth quotes
- Version 0.3 9:39pm After reading ¾ of Daniel Waxman's Unreasonable Effectiveness of Mathematics paper
    - Added Epistemic Tractability, Generality, Surprisingness, Discoverable by Theory Alone
    - Added Bourbaki mother-structures, correct definition of Langlands program
    - Added definitions of qualities of beauty, examples

11 July 2025

- Version 0.22 8:33pm After dinner, rearranged the order of Why Mathematics. Corrected formatting (removed empty pages).
- Version 0.21 6:11pm Some content additions (books to read, how Philosophy of Mathematics book was discovered)

- Version 0.2 6pm: Corrected formatting, reordered and rephrased sections
- Version 0.1 2pm: Corrected formatting, added Francis Su's book
- Version 0 9:30am: Drafted based on WhatsApp messages on 6 July (Sun) for Pebble (from 23 June request for motivation for Why Do Maths)
    - Will say next time I meet her that it is done because of marks and you can read the enrichment if you disagree and you are welcome to suggest other opinions for written discussion.

- **On Speed of Performance vs Mastery as Success Indicator in Mathematics**

    - "**I am a slow thinker**, and have to spend a lot of time before I can clean up my ideas and make progress." – Maryam Mirzakhani (Fields Medalist, in the book Mathematics for Human Flourishing Chp 8 Struggle)

    - "In spite of my success, I was always deeply uncertain about my own intellectual capacity; **I thought I was unintelligent**. And it is true that I was, and still am, **rather slow. I need time to seize things because I always need to understand them fully**." — Laurent Schwartz (Fields Medalist, in the book Mathematics for Human Flourishing Chp 8 Struggle)

    - "The **most profound contributions** to mathematics are often made by **tortoises rather than hares**" – Timothy Gowers (Fields Medalist, author of Mathematics A Very Short Introduction)
        - https://math.stackexchange.com/questions/8838/is-speed-an-important-quality-in-a-mathematician

- **On Mid-Life Career Changes**
    - "Of course you are happy, but deep down, you're a little bit worried that they might eventually figure out that you're not actually that good. I am a reasonably good mathematician, but am I Fields Medal-worthy?" – June Huh (2022 Field's Medal Winner)
        - https://www.quantamagazine.org/june-huh-high-school-dropout-wins-the-fields-medal-20220705/

#Todo

- Clean up examples of the qualities of beauty, applications and human interest sections
- Structure: Structure more
- Structure: Provide condensed (bold key ideas) and simplified explanations
- Examples: Easy, everyday if overlooked applications
- Examples: Analogies to jobs like being a detective
- Add comics (similar to Francis Su’s astronaut looking at a flower in Exploration chapter in Mathematics for Human Flourishing)
- Explain on website, YouTube